

HOW WELL DO STANDARD SOLAR MODELS DESCRIBE THE RESULTS OF SOLAR NEUTRINO EXPERIMENTS?*

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ABSTRACT. The neutrino fluxes calculated from the 14 standard solar models published recently in refereed journals are inconsistent with the results of the 4 pioneering solar neutrino experiments if nothing happens to the neutrinos after they are created in the solar interior. The calculated fluxes and the experimental results are in good agreement if neutrino oscillations occur.

1. Introduction

Solar neutrino research has achieved its primary goal, the detection of solar neutrinos, and is now entering a new phase in which large electronic detectors will yield vast amounts of diagnostic data. The new experiments (Arpesella et al. 1992, Takita 1993, McDonald 1994) will focus on testing the prediction of standard electroweak theory (Glashow 1961, Weinberg 1967, Salam 1968) that essentially nothing happens to electron type neutrinos after they are created by nuclear fusion reactions in the interior of the sun. The purpose of this talk is, on the eve of the new experiments, to assess the results of three decades of confrontation between solar models and solar neutrino experiments and to indicate some of the challenges that lie ahead.

The four pioneering experiments—chlorine (Davis 1964, 1994), which uses C_2Cl_4 as a detector, Kamiokande (Suzuki 1995), a water Cerenkov experiment, GALLEX (Anselmann et al. 1995), and SAGE (Abdurashitov et al. 1994), gallium radiochem-

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ical experiments—have all observed neutrino fluxes with intensities that are within a factors of a few of those predicted by standard solar models. Three of the experiments (chlorine , GALLEX, and SAGE) are radiochemical and each radiochemical experiment measures one number, the total rate at which neutrinos above a fixed energy threshold (which depends upon the detector) are captured. The sole electronic detector among the initial experiments, Kamiokande, has shown that the neutrinos come from the sun, by measuring the recoil directions of the electrons scattered by solar neutrinos, and has also demonstrated that the neutrino energies are in the range expected on the basis of the standard solar model.

Despite continual refinement of solar model calculations of neutrino fluxes over the past 35 years (see, e.g., the collection of reprint articles in Bahcall, Davis, Parker, Smirnov, and Ulrich 1995), the discrepancies between observations and calculations have gotten worse with time. All four of the pioneering solar neutrino experiments yield event rates that are significantly less than predicted by standard solar models. Moreover, there are well known inconsistencies between the different experiments if the observations are interpreted assuming that nothing happens to the neutrinos after they are created.

In this talk, I will first summarize the results of all the recently published standard solar model calculations and compare them with the results of the four solar neutrino experiments. For purposes of the summary, I will assume that, as implied by standard electroweak theory, nothing happens to the neutrinos after they are created. Then I will recall the results of many authors which show that the results of the solar neutrino experiments can be explained well if neutrinos oscillate between different eigenstates, i.e., between different types of neutrinos. Finally, I will discuss the implications for

astronomy of the neutrino experiments.

2. Observation versus Calculation

Figure 1 displays the calculated ${}^7\text{Be}$ and ${}^8\text{B}$ solar neutrino fluxes for all 14 of the standard solar models with which I am familiar that have been published in refereed science journals since 1988 (and before the cutoff date for this review: June 1, 1996). The first systematic discussion of the relation between helioseismology and solar neutrino research was published in 1988 (Bahcall and Ulrich 1988). I normalize the fluxes by dividing each published value by the flux from the most recent Bahcall and Pinsonneault (1995) standard solar model (hereafter BP95) which makes use of improved input parameters and includes heavy element and helium diffusion. The abscissa is the normalized ${}^8\text{B}$ flux and the numerator is the normalized ${}^7\text{Be}$ neutrino flux. The box shows the estimated 3σ uncertainties in the predictions of the standard solar model (BP95). The abbreviations that indicate references to individual models are identified in the caption of Figure 1.

All of the solar model results from different groups fall within the estimated 3σ uncertainties in the model predictions. This agreement between the results of 14 groups demonstrates the robustness of the predictions since the calculations use different computer codes and involve a variety of choices for the nuclear parameters, the equation of state, the stellar radiative opacity, the initial heavy element abundances, and the physical processes that are included. In fact, all published standard solar models give the same results for solar neutrino fluxes to an accuracy of better than 10% if the same input parameters and physical processes are included (Bahcall and Pinsonneault 1992, 1995)

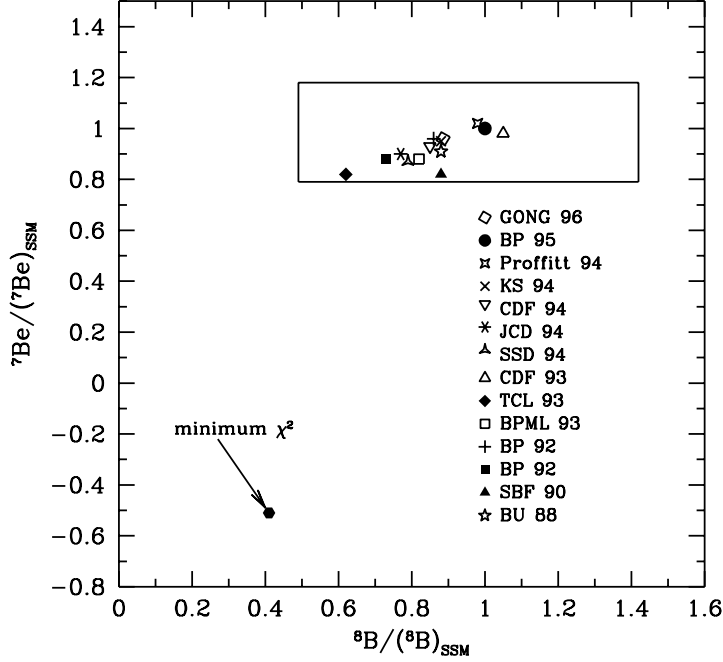


Fig. 1. The calculated ${}^7\text{Be}$ and ${}^8\text{B}$ solar neutrino fluxes for all 14 of the standard solar models. All of the fluxes have been normalized by dividing by the Bahcall and Pinsonneault (1995) standard solar model (SSM) values. The abbreviations of the various solar models are GONG (Christensen-Dalsgaard et al. 1996), BP 95 (Bahcall and Pinsonneault 1995), KS 94 (Kovetz and Shaviv 1994), CDF 94 (Castellani, Degl’Innocenti, Fiorentini, Lissia, and Ricci 1994), JCD 94 (Christensen-Dalsgaard 1994), SSD 94 (Shi, Schramm, and Dearborn 1994), CDF 93 (Castellani, Degl’Innocenti, and Fiorentini 1993), TCL 93 (Turck-Chièze and Lopes 1993), BPML 93 (Berthomieu, Provost, Morel, and Lebreton 1993), BP 92 (Bahcall and Pinsonneault 1992), SBF 90 (Sackman, Boothroyd, and Fowler 1990), and BU 88 (Bahcall and Ulrich 1988).

The largest contribution to the dispersion in values in Figure 1 is caused by the inclusion, or non-inclusion, of element diffusion in the stellar evolution codes. The Proffitt (1994), the Bahcall and Pinsonneault (1995), and the Christensen-Dalsgaard et al. (1996) models all include helium and heavy element diffusion. The predicted fluxes in these three models agree to within $\pm 10\%$, although the models are calculated using different mathematical descriptions of diffusion (and somewhat different input parameters),

The calculated value that is furthest from the center of the box is by Turck-Chièze and Lopes (1993), which does not include either helium or heavy element diffusion. However, the Turck-Chièze and Lopes best estimate is still well within the 3σ box.

Helioseismology has recently sharpened the disagreement between observations and the predictions of solar models with standard (non-oscillating) neutrinos. By including element diffusion, four solar models near the center of the box in Figure 1 (models of Bahcall and Pinsonneault 1992, Proffitt 1994, BP95, and Christensen-Daalsgard et al. 1996) yield values for the depth of the convective zone and the primordial helium abundance that are in agreement with helioseismological measurements. (The model of Richard et al. 1996 yields results in good agreement with the four solar models just mentioned that include element diffusion, but was not yet published in *Astron. and Astrophys.* by the cutoff date, June 1, 1996.)

Solar models that do not include diffusion are not consistent with the helioseismological evidence (see discussion in Christensen-Dalsgaard, Proffitt, and Thompson 1993, Guzik and Cox 1993, BP95, and Christensen-Daalsgard et al. 1996). The results of the major new helioseismological initiatives, GONG and SOHO, will provide important additional constraints on the solar models.

In my view, only solar models that include element diffusion should, in the future, be called “standard solar models”. These “standard models” all lie close to the center of the rectangular error box in Figure 1. The physics of diffusion is simple and there is an exportable subroutine available for calculating diffusion in stars (see <http://www.sns.ias.edu/~jnb>). Observation requires, and computing technology easily permits, the inclusion of diffusion in any standard stellar evolution code.

How do the observations from the four pioneering solar neutrino experiments agree with the solar model calculation? Plamen Krastev and I (see Bahcall and Krastev 1996 for a description of the techniques) have recently compared the predicted standard model fluxes, with their estimated uncertainties, and the observed rates in the chlorine, Kamiokande, GALLEX, and SAGE experiments. The theoretical solar model and experimental uncertainties, as well as the uncertainties in the neutrino cross sections, have been combined quadratically. Using the predicted fluxes from the BP95 model, the χ^2 for the fit to the four experiments is

$$\chi_{\text{SSM}}^2(\text{all 4 experiments}) = 56 . \quad (1)$$

The theoretical uncertainties (from the solar model and the neutrino cross section calculations) and the experimental errors (statistical and systematic) have been combined quadratically in obtaining equation (1).

Suppose we now ignore what we have learned from solar models and allow the important ${}^7\text{Be}$ and ${}^8\text{B}$ fluxes to take on any non-negative values. What is the minimum value of χ^2 for the 4 experiments, when the only constraint on the fluxes is the requirement that the luminosity of the sun be supplied by nuclear fusion reactions among light elements? We include the nuclear physics inequalities between neutrino fluxes (see section 4 of Bahcall and Krastev 1996) that are associated with the luminosity constraint and maintain the standard value for the almost model-independent ratio of pep to pp neutrinos.

The best fit for arbitrary ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes is obtained for ${}^7\text{Be}/({}^7\text{Be})_{\text{SSM}} =$

0 and ${}^8\text{B}/({}^8\text{B})_{\text{SSM}} = 0.40$, where

$$\chi^2_{\text{minimum}}(\text{all 4 experiments; arbitrary } {}^7\text{Be}, {}^8\text{B}) = 14.4 . \quad (2)$$

The CNO neutrinos were assumed equal to their standard model values in the calculations that led to Eq. 2. The fit can be further improved if we set the CNO neutrino fluxes equal to zero. Then, the same search for arbitrary ${}^7\text{Be}$ and ${}^8\text{B}$ neutrino fluxes leads to

$$\chi^2_{\text{minimum}}(\text{all 4 experiments; arbitrary } {}^7\text{Be}, {}^8\text{B}; \text{CNO} = 0) = 5.9 . \quad (3)$$

If we drop the physical requirement that the fluxes be positive definite, the minimum χ^2 occurs (cf. Figure 1) for a negative value of the ${}^7\text{Be}$ flux; this unphysical result is a reflection of what has become known in the physics literature as “the missing ${}^7\text{Be}$ solar neutrinos.”. The reason that the ${}^7\text{Be}$ neutrinos appear to be missing (or have a negative flux) is that the two gallium experiments, GALLEX and SAGE, have an average event rate of 74 ± 8 SNU, which is fully accounted for in the standard model by the fundamental $p - p$ and pep neutrinos (best estimate 73 ± 1 SNU). In addition, the ${}^8\text{B}$ neutrinos that are observed in the Kamiokande experiment will produce about 7 SNU in the gallium experiments, unless new particle physics affects the neutrinos.

To me, these results suggest strongly that the assumption on which they are based—nothing happens to the neutrinos after they are created in the interior of the sun—is incorrect. A less plausible alternative (in my view) is that some of the experiments are wrong; this must be checked by further experiments.

3. Are Neutrino Oscillations the Answer?

In the simplest version of the standard model of electroweak interactions (Glashow 1961, Weinberg 1967, Salam 1968), electron-type neutrinos that are created in the center of the sun by nuclear fusion reactions remain electron-type neutrinos as they pass through the solar material and propagate to detectors on earth. The three radiochemical experiments (chlorine, GALLEX, and SAGE) are sensitive only to electron-type neutrinos, whereas Kamiokande has reduced sensitivity also for muon or tau neutrinos.

Particle physicists have proposed a number of possible solutions to the problem posed by the discrepancy between solar neutrino observations and the combined standard predictions of solar models and electroweak theory. The most popular of these solutions involve neutrino oscillations in vacuum (Pontecorvo 1968) and matter enhanced resonant neutrino oscillations, the so-called MSW effect (Wolfenstein 1978, Mikheyev and Smirnov 1985).

The comparison between theory and observations is improved significantly if neutrino oscillations occur. I give here the results of calculations for the particle physics solutions that are most frequently discussed in the physics literature. The minimum χ^2 obtained with two degrees of freedom (mixing angle, and difference of squared masses) is (Bahcall and Krastev 1996), for the most-popular small mixing angle Mikheyev-Smirnov-Wolfenstein (MSW) solution,

$$\chi_{\min}^2 = 0.31 \quad , \quad \text{SMA} \quad . \quad (4)$$

For the large mixing angle (MSW) solution,

$$\chi_{\min}^2 = 2.5 \quad , \quad \text{LMA} . \quad (5)$$

For vacuum neutrino oscillations,

$$\chi_{\min}^2 = 2.5 \quad , \quad \text{vacuum oscillations} . \quad (6)$$

Neutrino oscillations provide a significant improvement in the minimum χ^2 for the four operating solar neutrino experiments.

4. Discussion

The combined predictions of the standard solar model and the standard electroweak theory disagree with the results of the four pioneering solar neutrino experiments. Comparing the combined predictions to the existing data, we obtain values for χ_{standard}^2 of ~ 56 . The fits are much improved if neutrino oscillations, which are described by two free parameters, are included in the calculations. With neutrino oscillations, the characteristic value for $\chi_{\min, \text{osc.}}^2 \sim 1$. New experiments (Arpesella et al. 1992, Takita 1993, McDonald 1994) involving large electronic detectors of individual neutrino events will decide in the next few years if neutrino oscillations are indeed important in interpreting solar neutrino experiments.

For astrophysics, the most important quantities that can be deduced from neutrino oscillation experiments are the neutrino mass differences (Only the squares of mass differences appear in the oscillation equations, since the propagation phases are determined by the squares of masses.). For the currently most popular oscillation scenario, the MSW effect (Mikheyev and Smirnov 1985; Wolfenstein 1978) [which involves resonant flavor

conversion in matter], the values of the mass differences reported in the literature are obtained by solving the differential equations for neutrino propagation in matter.

There is a simple analytic argument which allows one to estimate the neutrino masses that result from numerical solutions of the MSW propagation equations and to understand why the neutrino masses are given robustly by MSW theory. Let n_e , θ_V , Δm^2 , and E_ν be, respectively, the electron number density, the mixing angle in vacuum between two types of neutrino states (e.g., electron type and muon type), the difference of the squared masses of the two different neutrino types, and the neutrino energy. Then one can show analytically (Mikheyev and Smirnov 1985) that there is a resonance in the neutrino propagation only if somewhere in the sun the electron density at resonance satisfies the following numerical equation (Eq. 9.53 of Bahcall 1989):

$$\frac{n_e(\text{resonance})}{n_e(\text{center of sun})} = 0.7 \cos 2\theta_V \left[\frac{\Delta m^2}{10^{-5} \text{eV}^2} \right] \left[\frac{1 \text{ MeV}}{E_\nu} \right]. \quad (7)$$

Obviously, there is no solution to Eq. (7) if the required value for $n_e(\text{resonance})$ exceeds the highest value of the electron density, which occurs at the center of the sun.

As remarked in Section 2, the two gallium experiments suggest that the $p - p$ neutrinos (with energies less than 0.4 MeV) are not affected by resonance oscillations while the 0.86 MeV ${}^7\text{Be}$ neutrinos are affected by the resonance. Requiring that $n_e(\text{resonance})/n_e(\text{center of sun})$ be greater than unity for $E_\nu = 0.4$ MeV and less than unity for $E_\nu = 0.9$ MeV, yields

$$\Delta m^2 \sim 10^{-5} \text{ eV}^2. \quad (8)$$

It is plausible to suppose that Eq. (8) gives approximately the mass of the muon neutrino (i.e., $m(\nu_\mu) \sim 0.003$ eV), which is expected to be heavier than the electron neutrino. Many particle physics models suggest that the mass of the tau neutrino is larger than the mass of the muon neutrino by a factor whose order of magnitude is the ratio of the square of the mass of the top quark (176 GeV) to the square of the mass of the charmed quark (1.6 GeV). One might anticipate, therefore, a mass for the tau neutrino that is within a factor of ten of $10^4 \times 0.003$ eV, or

$$m(\nu_\tau) \sim 10^{1.5} \text{ eV}. \quad (9)$$

This mass for the tau neutrino would be cosmologically important, potentially containing enough dark matter to close the universe.

Finally, we may ask: What have solar neutrino experiments taught us about astronomy? The operating experiments have achieved the primary goal of solar neutrino astronomy by showing empirically that the sun shines via nuclear fusion reactions. Moreover, the observed and the standard predicted neutrino interaction rates agree within factors of a few, providing (see below) semi-quantitative confirmation of the calculation of temperature-sensitive nuclear fusion rates in the solar interior.

The important ^8B neutrino flux depends upon the central temperature of the sun as approximately T^{24} (Bahcall and Ulmer 1996). The maximum range allowed by neutrino oscillation scenarios and the results of the four operating solar neutrino experiments is (Bahcall and Krastev 1996):

$$0.37 \leq \phi(^8\text{B})/\phi(^8\text{B})_{\text{SSM}} \leq 2.84 . \quad (10)$$

Thus the Kamiokande experiment constrains the total ^8B neutrino flux to be within a factor of three of the value predicted by standard solar models (if neutrino oscillations, vacuum or resonant matter oscillations are occurring).

The possibility that neutrino oscillations are occurring complicates greatly the interpretation of solar neutrino data. Until new experiments are performed, one cannot even rule out empirically an *ad hoc* scenario (Bahcall, Fukugita, and Krastev 1996), not predicted by any detailed solar model, in which the sun shines by CNO rather than p - p fusion reactions.

The SNO (McDonald 1994) heavy water experiment will measure for ^8B solar neutrinos both the total flux and the flux of electron type neutrinos. The Superkamiokande ultrapure water experiment (Takita 1993), which began operating April 1, 1996, is primarily sensitive to electron type neutrinos but has some sensitivity to other neutrino types also. The results of these experiments will determine the absolute value of the ^8B neutrino production rate in the sun, which was the originally-stated purpose of the chlorine experiment (Bahcall 1964, Davis 1964) before the complications due to possible new neutrino physics were recognized. The results from these new experiments will constitute a critical, quantitative test, independent of uncertainties about new particle physics, of solar model calculations of nuclear fusion rates in the center of the sun.

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